Product Analysis: Interest Rate Product Structures

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Par-Coupon Yield Curves

Internal Rate of Return

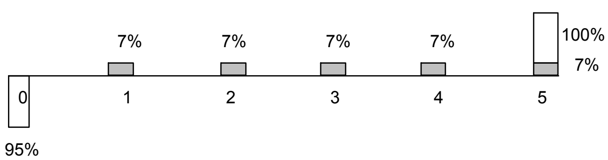
All bond yields are expressed in terms of internal rate of return (IRR). In the special case where the IRR equals the coupon rate (the coupon divided by the principal), the yield is called a **par-coupon yield**.

Internal rate of return is calculated easily using a financial calculator. IRR might be defined as follows: the specific rate of discount which returns a present value equal to the bond's price when applied to each of its cash flows, compounded over the relevant number of periods for each cash flow.

Calculators cannot solve for IRR directly. They find it by trying values over and over until they calculate a present value equal to the given price. This method of calculating is called *iterative*. IRR is an iterative result.

Example

What is the IRR of the following bond?



There is only one discount rate (I%) which makes the following equation true:

Finding the right value for I% by trying lots of possible values—the method used by the calculator—is not easy. Far easier is to let the calculator do the work:

Value	Key	Display
5	[N]	5.0000
95	[CHS][PV]	–95.0000
7	[PMT]	7.0000
100	[FV]	100.0000
	[I%]	8.2609

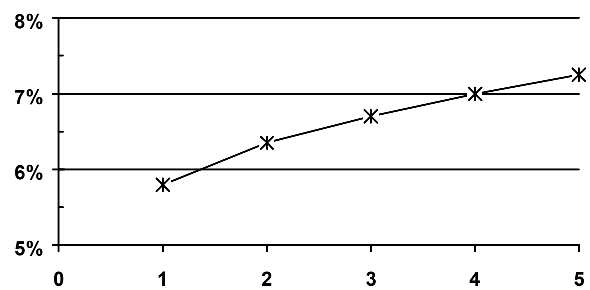
The IRR or yield to maturity of the above bond is 8.2609%.

If the price of the above bond is 100%, its IRR or yield to maturity is 7%, as shown above. The price equation is the same:

$$100\% = \frac{7\%}{(1+I\%)^1} + \frac{7\%}{(1+I\%)^2} + \frac{7\%}{(1+I\%)^3} + \frac{7\%}{(1+I\%)^4} + \frac{7\%}{(1+I\%)^5} + \frac{100\%}{(1+I\%)^5}$$

The only value for I% which returns a price of 100% is 7%.

Setting up the Par-Coupon Yield Curve



The yield curve below reflects the IRR for bonds of the stated maturity having a price of 100%.

	Par-Coupon Yields
0	
1	5.80%
2	6.35%
3	6.70%
4	7.00%
5	7.25%

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Markets and Par-Coupon Rates: Importance of the Swap Market

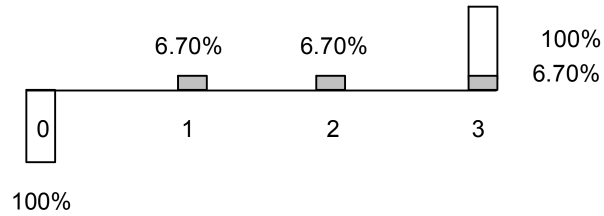
The par-coupon yield curve above is a highly unique yield curve. It almost never exists in any of the world's bond markets, since bonds tend to have coupons set in even multiples of 1/4%, and thus even newly issued bonds have prices which are close to par, but never exactly par.

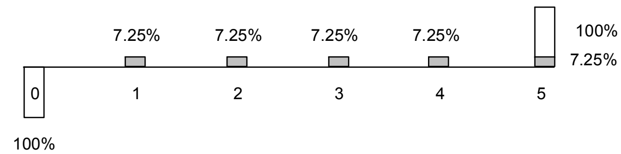
There is one market, however, which trades by changing the **yield** at each maturity as market expectations change, thus keeping the price at 100%: the interest rate swap market. Interest rate swap quotes are always par-coupon yields. To obtain a good par-coupon curve for any currency with an interest rate swap market, swap rates are ideal.

Problems Using IRR to Value Securities

There is a fundamental problem with using IRR, or par-coupon rates in general, to value securities. IRR assumes that the present value of each cash flow in a security is calculated using the same interest rate. IRR also assumes that every cash flow paid or received in a security can be reinvested (or borrowed) at the same rate. This ignores the fact that the yield curve is almost never flat, as discussed above in the previous section, and reflected in the above par-coupon yield curve.

Even more problematic, different IRRs assume that cash flows paid or received **on the same date** but in two different instruments are valued using different rates, according to which instrument they are in. Compare the following two bonds taken from the par-coupon curve above:





The three-year bond's price can be calculated as follows:

$$100\% = \frac{6.70\%}{(1+6.70\%)^1} + \frac{6.70\%}{(1+6.70\%)^2} + \frac{6.70\%}{(1+6.70\%)^3} + \frac{100\%}{(1+6.70\%)^3}$$

The five-year bond's price is calculated in the same way:

$$100\% = \frac{7.25\%}{(1+7.25\%)^1} + \frac{7.25\%}{(1+7.25\%)^2} + \cdots + \frac{7.25\%}{(1+7.25\%)^5} + \frac{100\%}{(1+7.25\%)^5}$$

Coupons paid at the end of year one are valued at either 6.70%, if they occur in the 3-year bond, or 7.25%, if they occur in the 5-year bond. Even worse, the first coupons in all of the par-coupon bonds along the yield curve, which all occur on the very same day, are valued at five different rates—depending on which instrument they are in.

There must be a more accurate way to price securities. There is.

Zero-Coupon Yield Curves

Par-coupon curves give a good representation of where market yields lie. But as explained above, they are not very good for calculating prices. The market prefers to use **a single rate** (rather than a whole collection of different rates) to value cash flows that occur on the same day. It makes no difference which maturity the instrument has which the cash flow is in, its value, its price in the market, should be calculated using the same rate used for every other cash flow occurring on that date.

This curve of pricing rates is the zero-coupon yield curve. And it can be easily derived from the par-coupon rates in the market.

Deriving the Zero-Coupon Yield Curve

Each of the cash flows in the above par-coupon bonds can be valued using a zero-coupon rate. To begin, it is necessary to have one par-coupon instrument that is also a zero-coupon structure. In the simplified yield curve above, that is the one-year par-coupon rate:

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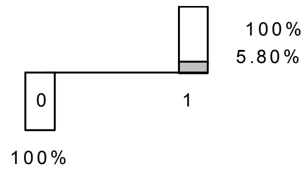


Since there is no interim coupon, the single par coupon paid at the end of Year 1 together with the principal repayment, constitute, in effect, a zero-coupon structure.

The zero-coupon rate can be calculated by solving the equations introduced above for I%:

$$100\% = \frac{105.80\%}{(1+I\%)^1}$$

$$I\% = \frac{105.80\%}{100\%} - 1$$

$$I\% = 5.80\%$$

Or, using the financial calculator:

Value	Key	Display
1	[N]	1.0000
100	[CHS][PV]	–100.0000
0	[PMT]	0.0000
100	[FV]	100.0000
	[I%]	5.8000

Where there is only a single coupon, the par-coupon rate is equal to the zero-coupon rate. 5.80% is both a zero-coupon rate and a par-coupon rate. In the above equation, it is used as a zero-coupon rate.

Using Present Value Factors

Calculating a price using zero-coupon rates is clumsy because it is necessary to divide by one plus the rate:

$$PV = \frac{\text{Cash Flow}}{(1+I\%)^n}$$

$$1 = \frac{1.0580}{(1+0.0580)^1}$$

This calculation can be simplified by using a **present value factor** in place of dividing by one plus the rate.

Modifying the above equation using algebra defines present value factors:

$$PV = \text{Cash Flow} \times \frac{1}{(1+I\%)^n}$$

$$1 = 1.0580 \times \frac{1}{(1+0.0580)^1}$$

$\frac{1}{(1+0.0580)^1}$ is the present value factor for one year.

The general definition is:

$$PVf = \frac{PV}{FV}$$

Deriving Present Value Factors

The present value factors for 2, 3, 4 and 5 years can be derived successively using the par-coupon rates and all previously derived PV factors.

Year 1

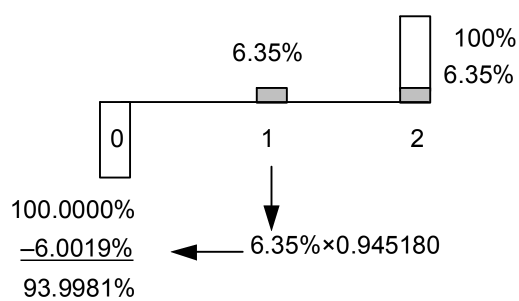
The PV factor for Year 1 is straightforward, as both the PV and FV are known:

$$PVf_1 = \frac{PV}{FV} = \frac{1}{1.0580} = 0.945180$$

Year 2

The PV factor for Year 2 is calculated using the 2-year par-coupon rate of 6.35% and the 1-year PV factor calculated above. The first step is to create a 2-year zero-coupon cash flow by "stripping" away the first coupon of 6.35% at the end of Year 1. The second step is to calculate the 2-year PV factor.

1. Calculate the present value of the first coupon using the 1-year PV factor calculated above:



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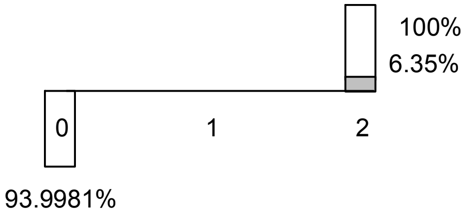
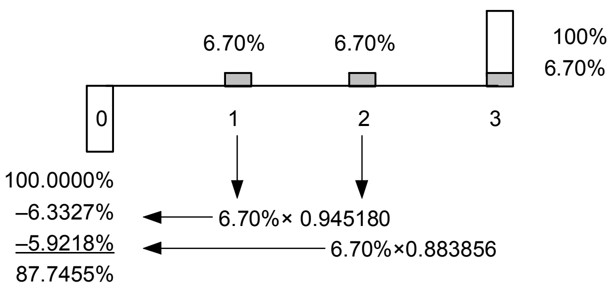
The present value of the first coupon is 6.0019%. Subtracting it from the original price of 100% gives the present value of the cash flows at Year 2: 93.9981%:



The PV factor for Year 2 is now easily calculated as above:

$$PVf_2 = \frac{PV}{FV} = \frac{0.939981}{1.0635} = 0.883856$$

Years 3-5
The same process is followed for each successive year, using the PV factors calculated previously:



$$PVf_3 = \frac{PV}{FV} = \frac{0.877455}{1.067} = 0.822357$$

This process can be summarised in an algebraic formula as follows:

$$PVf_n = \frac{PV}{FV} = \frac{1 - PMT \times \sum_{t=1}^{n-1} PVf_t}{1 + PMT}$$

where:

- PVf_n = Present value factor at any specific date "n"
- PMT = The par coupon level for the respective maturity
- t = A counter to show that there is a PV factor for each coupon date through the maturity

This formula might be read as "The present value factor at time 'n' is equal to one minus the coupon times the sum of the present value factors for all previous coupon dates, divided by 1 plus the coupon."

Year 3

$$PVf_3 = \frac{PV}{FV} = \frac{1 - .067 \times (0.945180 + 0.883856)}{1 + .067} = 0.822357$$

Year 4

$$PVf_4 = \frac{PV}{FV} = \frac{1 - .07 \times (0.945180 + 0.883856 + 0.822357)}{1 + .07} = 0.761124$$

Year 5

$$PVf_5 = \frac{PV}{FV} = \frac{1 - .0725 \times (0.945180 + 0.883856 + 0.822357 + 0.761124)}{1 + .0725} = 0.701718$$

Zero-Coupon Yield Curve

The PV factors represent the price of a zero-coupon cash flow of 1 at each date. They thus give the respective zero-coupon rates very easily.

The simplified general formula is:

$$I\%_{zero-coupon_n} = \left(\frac{1}{PVf_n}\right)^{\frac{1}{n}} - 1$$

This formula might be read as "The zero-coupon rate for a future date 'n' is equal to one divided by the PV factor for that date, raised to the power of one divided by the number of periods, minus one."

Using this formula the zero-coupon curve can be calculated algebraically:

$$I\%_{zero_1} = \left(\frac{1}{0.945180}\right)^{\frac{1}{1}} - 1 = 5.80\%$$

$$I\%_{zero_2} = \left(\frac{1}{0.883856}\right)^{\frac{1}{2}} - 1 = 6.3676\%$$

$$I\%_{zero_3} = \left(\frac{1}{0.822357}\right)^{\frac{1}{3}} - 1 = 6.7366\%$$

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$$I\%_{zero_4} = \left(\frac{1}{0.761124}\right)^{\frac{1}{4}} - 1 = 7.0622\%$$

$$I\%_{zero_5} = \left(\frac{1}{0.701718}\right)^{\frac{1}{5}} - 1 = 7.3415\%$$

The zero-coupon rates can also be calculated using the financial keys of the calculator:

Key	Year 1	Year 2	Year 3	Year 4	Year 5
[N]	1.0000	2.0000	3.0000	4.0000	5.0000
[PV]	– 0.945180	– 0.883856	– 0.822357	– 0.761124	– 0.701718
[PMT]	0.0000	0.0000	0.0000	0.0000	0.0000
[FV]	1.0000	1.0000	1.0000	1.0000	1.0000
[I%]	5.8000	6.3676	6.7366	7.0622	7.3415

With the addition of the PV factors and zero-coupon rates, the yield curve is becoming more useful. If the cash flows of an instrument are known, its price in the current market environment can be calculated using PV factors. Even if the cash flows do not occur on the annual payment dates, interpolation can produce the appropriate PV factor.

The yield curve introduced earlier can now be expressed more fully:

	Par-Coupon	PV Factors	Zero-Coupon
0		1.000000	
1	5.80%	0.945180	5.8000%
2	6.35%	0.883856	6.3676%
3	6.70%	0.822357	6.7366%
4	7.00%	0.761124	7.0622%
5	7.25%	0.701718	7.3415%

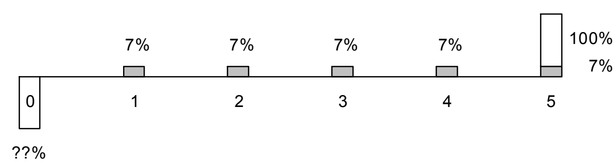
Non-Par Coupon Bonds

PV factors can be used to price any instrument using current market rates. They are thus the ideal mechanism for *marking to market* securities issued earlier. "Marking to market," means calculating the price of an instrument in the current market environment.

The yield curve developed so far is:

	Par-Coupon	PV Factors	Zero-Coupon
0		1.000000	
1	5.80%	0.945180	5.8000%
2	6.35%	0.883856	6.3676%
3	6.70%	0.822357	6.7366%
4	7.00%	0.761124	7.0622%
5	7.25%	0.701718	7.3415%

This yield curve can be used to price the following bond:



Each cash flow is priced using the appropriate PV factor:

Cash Flow	PV Factor	Present Value
7 ×	0.945180 =	6.616260
7 ×	0.883856 =	6.186992
7 ×	0.822357 =	5.756499
7 ×	0.761124 =	5.327868
107 ×	0.701718 =	75.083826
	TOTAL	**98.971445**

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Forward Yield Curves

In addition to information about rates for periods beginning today, yield curves provide information about rates for periods beginning in the future.

So-called "forward rates" are used to evaluate refinancing decisions, to lock in investment rates or borrowing costs for projects which will only be undertaken in the future, to take views about where the yield curve might be at a forward date, and to price derivative products.

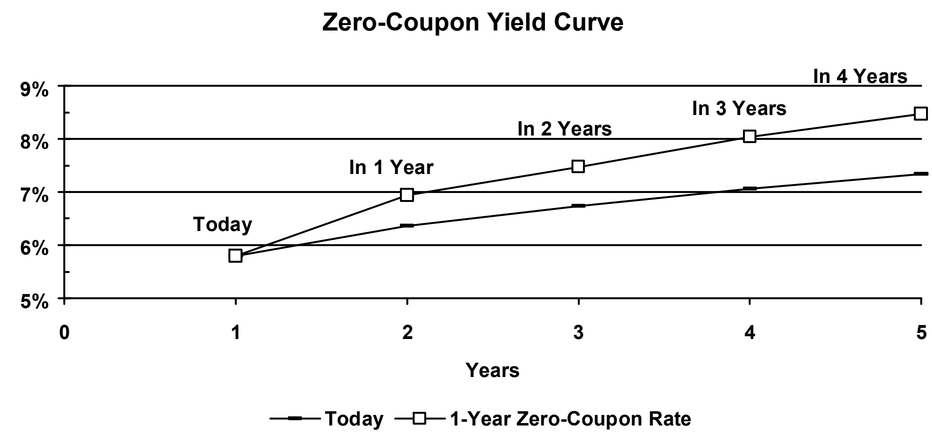
Forward curves—like yield curves starting today—come in both zero-coupon and par-coupon form. In addition, market participants often look at forward rates for both par-coupon and zero-coupon cash flow structures in two guises:

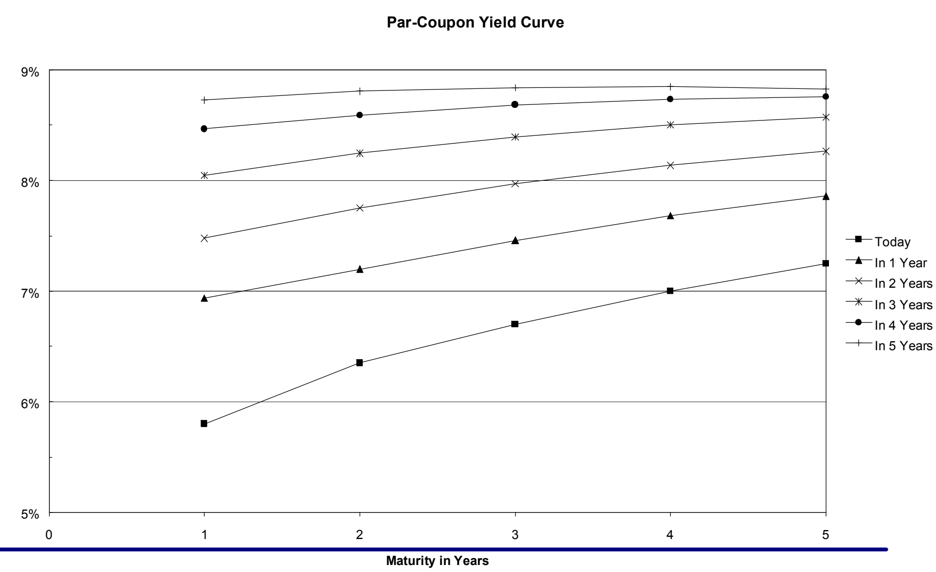
1. The same rate (the 1-year rate, for example) at a series of forward dates.
2. The entire yield curve at a series of forward dates.

These two varieties of forward curves are explained below.

The same rates (the 1-year rate, for example) at a series of forward dates.

The yield curve below shows the zero-coupon yield curve, i.e. the zero-coupon rates for one, two, three, four and five years beginning today. It also shows the 1-year zero-coupon rate beginning today, the 1-year zero-coupon rate beginning in one year (and ending in two years), the 1-year zero-coupon rate beginning in two years, etc.





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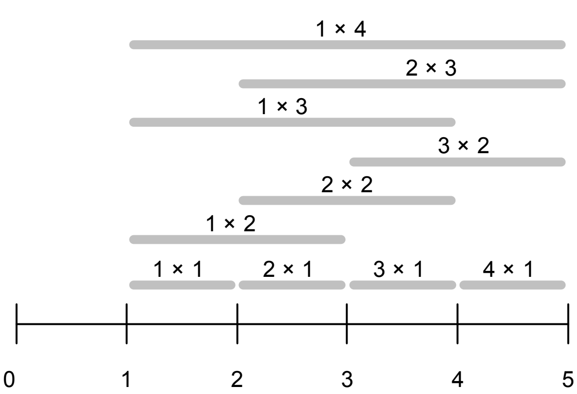
The 1-year rate beginning in 1 year is plotted above 2 years. Its ending date—2 years from now—is the same date as the ending date of today's 2-year zero-coupon rate. Thus they are plotted above the same date. Market participants often plot a series of forward rates having the same tenor in this manner, since each rate plotted is for a different forward period.

The entire yield curve at a series of forward dates. The yield curve above shows the one-, two-, three-, four- and five-year par-coupon rates today. It also shows the one-, two-, three-, and four-year par-coupon rates beginning in one year; the one-, two- and three-year par-coupon rates beginning in two years; the one- and two -year par-coupon rates beginning in three years; and the one-year par-coupon rate beginning in four years.

Unlike the first graph above, the entire yield curve at each forward date is plotted as if it begins today. This is because this graph is intended to show the evolution of the entire yield curve. Market participants often plot entire forward yield curves in this manner, as it highlights the evolution of the curve more vividly.

Over the next five years there are many forward periods. There is a forward rate for each of these periods:



The periods shown above are referred to in the market with specific names: the "one into one," which means the 1-year rate 1 year forward; the "2 into 1," or the 1-year rate 2 years forward; the "1 into 2," or the 2-year rate 1 year forward; the "2 into 3," or the 3-year rate 2 years forward; etc.

In a five-year period, these are the only full-year possibilities (assuming periods that begin and end on the annual anniversary date). The ending date of a forward period the farthest possible into the future must be at 5 years.

Calculating Forward Rates

Now you have enough information to know how to calculate forward rates. The fancy term is the Expectations Hypothesis of the Term Structure (EHT) or the No-Arbitrage Principle (or perhaps the no-free-lunch principle). In simple terms this means that the real world is full of reasonably smart people with access to reasonably the same information so it is reasonable to assume the term structure of interest rate (interest rates plotted against time –like the zero curve) creates no opportunities for people to simultaneously buy and sell interest rate products from different parts of the zero curve and instantly make money. Arbitrage is that simultaneous buying and selling. The assumption is that: if you could do it, then everybody would, and prices would soon adjust to remove the arbitrage opportunity.

Let's say there happens to be two used record shops directly across the street from each other, both with signs detailing their Yardbirds album specials. One shop is selling the "complete officially recommended" Yardbirds record set (6 albums) for $35 and the other shop is buying individual Yardbirds albums for $8. Now you're only interested in the BBC Sessions, so after a little thought you'd buy the set for $35, walk across the street and sell the other four albums for $32, and keep the BBC Sessions for a cost of $3. Now if you had sold all six, you would have made $13 – and that would be arbitrage. If the guy buying the records doesn't adjust his price quickly, he is going to have an overstock of over priced so-so Yardbirds albums very quickly (they were a great band, but produced a lot of so-so records).

The same is true for interest rate products. Let's say you had $100 to invest for 2 years, if you want to do all you're investing today, you could either:

Invest in a 2 year zero coupon instrument, or invest in a 1 year zero coupon instrument and get a guaranteed 1-year zero-coupon rate for the second year.

Using our example rates this is what you'd make:

$$100x\,(1+ 6.3676\%)^2 = 113.1407$$

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$$100\text{x } (1+ 5.8\%) \text{ x } (1 + Z_{1x1}) = FV$$

So if somebody was going to set the Z_{1x1} rate today, what would they set it to? Now in equation (2) you already know what FV equals, because the no free lunch principle tells you that it has to be 113.1407. If it wasn't, somebody could figure out how to take advantage of the zero curve to get a free lunch. If the rate Z_{1x1} produced a return higher than 113.1407, then that would be a lot like our hapless record store owner above.

So now you have an equation with only one known, which can be easily solved for.

$$105.80 \text{ x } (1 + Z_{1x1}) = 113.1407$$
$$Z_{1x1} = (113.1407 / 105.8) - 1$$
$$= 6.93824\%$$

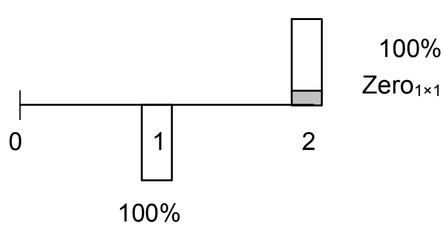
The next two sections show how to derive both zero-coupon and par-coupon forward rates.

Forward Zero-Coupon Yield Curve

Zero-coupon forward rates are calculated using the PV factor for the beginning and ending dates of the forward period. The zero-coupon rates and PV factors shown on page 5 above are:

	PV Factors	Zero-Coupon
0	1.000000	
1	0.945180	5.8000%
2	0.883856	6.3676%
3	0.822357	6.7366%
4	0.761124	7.0622%
5	0.701718	7.3415%

The 1×1 zero-coupon rate, for example, might be pictured as follows:

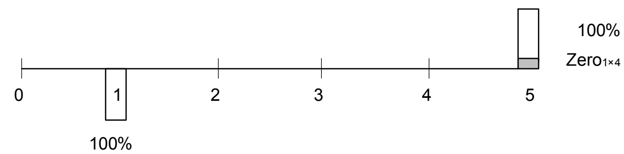


Using PV factors, the PV today of the 100% one year from now is set equal to the PV today of the 100% plus $Zero_{1\times1}$ two years from now:

$$100\% \times 0.945180 = \left(100\% + Zero_{1\times1}\right) \times 0.883856$$

$$Zero_{1\times1} = \frac{(0.945180 - 0.883856)}{0.883856} = \frac{0.945180}{0.883856} - 1 = 6.9382\%$$

To calculate a zero-coupon rate for a period greater than one year, it is necessary to "de-compound" the rate down to a 1-year rate. The 1×4 zero-coupon rate, for example, might be pictured like this:



Using PV factors, the PV today of the 100% one year from now is set equal to the PV today of the 100% plus $Zero_{1\times4}$ five years from now:

$$100\% \times 0.945180 = \left(100\% + Zero_{1\times4}\right) \times 0.701718$$

$$Zero_{1\times4} = \frac{(0.945180 - 0.701718)}{0.701718} = \frac{0.945180}{0.701718} - 1 = 34.6951\%$$

34.6951% is the rate for the full four-year period. To express this rate in the usual annual effective manner, it must be de-compounded:

$$Zero_{1\times4} = \left(\frac{0.945180}{0.701718}\right)^{\frac{1}{4}} - 1 = 7.7303\%$$

The general formula is similar to the formula for a zero-coupon rate shown above:

$$I\%_{a\times b} = \left(\frac{PVf_a}{PVf_{a+b}}\right)^{\frac{1}{b}} - 1$$

This formula might be read as "The forward zero-coupon rate for the *a* into *b* forward period, i.e. the period beginning *a* years from now and running for *b* years, is equal to the PV factor for the beginning date *a* divided by the PV factor for the ending date *a* + *b*, raised to the power of 1 divided by the number of years in the forward period *b*, less 1."

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This formula can be used to solve for all of the possible full-year forward zero-coupon rates as follows:

1×1

$$I\%_{1\times 1} = \left(\frac{0.945180}{0.883856}\right)^{\frac{1}{1}} - 1 = 6.9382\%$$

2×1

$$I\%_{2\times 1} = \left(\frac{0.883856}{0.822357}\right)^{\frac{1}{1}} - 1 = 7.4784\%$$

3×1

$$I\%_{3\times 1} = \left(\frac{0.822357}{0.761124}\right)^{\frac{1}{1}} - 1 = 8.0451\%$$

4×1

$$I\%_{4\times 1} = \left(\frac{0.761124}{0.701718}\right)^{\frac{1}{1}} - 1 = 8.4658\%$$

1×2

$$I\%_{1\times 2} = \left(\frac{0.945180}{0.822357}\right)^{\frac{1}{2}} - 1 = 7.2080\%$$

2×2

Intentionally left blank

3×2

$$I\%_{3\times 2} = \left(\frac{0.822357}{0.701718}\right)^{\frac{1}{2}} - 1 = 8.2552\%$$

1×3

Intentionally left blank

2×3

$$I\%_{2\times 3} = \left(\frac{0.883856}{0.701718}\right)^{\frac{1}{3}} - 1 = 7.9957\%$$

1×4

$$I\%_{1\times 4} = \left(\frac{0.945180}{0.701718}\right)^{\frac{1}{4}} - 1 = 7.7303\%$$

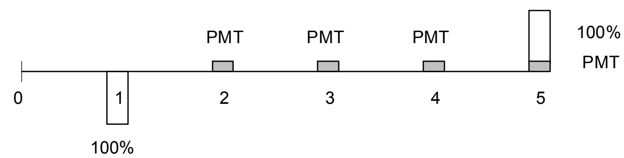
All of the rates above are forward zero-coupon rates.

Forward Par-Coupon Yield Curve

To calculate refinancing possibilities in the future, it is necessary to calculate forward par-coupon yields.

Calculating par-coupon rates for forward periods is done in a similar way to that shown above for zero-coupon yields, except that the coupon payments made during the forward period have to be figured in.

The 1×4 par-coupon rate, for example, might be pictured as follows:



"PMT" in the above picture refers to the coupon payment. Since the price of the bond is set to 100%, the coupon must be equal to the yield to maturity. "PMT" thus refers to **both** the coupon **and** the IRR.

Using PV factors, the PV today of the 100% one year from now is set equal to the PV today of the rest of the cash flows:

$$100\% \times 0.945180 = PMT \times \sum_{t=2}^{5} PVf_t + 100\% \times 0.701718$$

This equation says that the price of the par-coupon security for the forward period beginning one year from now and ending five years from now is 94.1580% today. This is set equal to the rest of the cash flows.

The PV today of the series of forward coupon payments is equal to $PMT \times \sum_{t=2}^{5} PVf_t$. This is just a shorthand way to say that each PMT is multiplied by its respective PV factor and then they are all added up.

Finally, the 100% at maturity is worth 70.1718% today.

Using algebra to rearrange the above equation to solve for PMT:

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$$PMT = \frac{(0.945180 - 0.701718)}{(0.883856 + 0.822357 + 0.761124 + 0.701718)} = 7.6825\%$$

The general formula for a forward par-coupon rate "a" years forward running for "b" years is:

$$PMT_{a \times b} = \frac{PVf_a - PVf_{a+b}}{\sum_{t=a+1}^{a+b} PVf_t}$$

This formula can be used to solve for all of the possible full-year forward par-coupon rates as follows:

1×1

$$I\%_{1\times1} = \frac{0.945180 - 0.883856}{0.883856} = 6.9382\%$$

2×1

$$I\%_{2\times1} = \frac{0.883856 - 0.822357}{0.822357} = 7.4784\%$$

3×1

Intentionally left blank

4×1

$$I\%_{4\times1} = \frac{0.761124 - 0.701718}{0.701718} = 8.4658\%$$

1×2

$$I\%_{1\times2} = \frac{0.945180 - 0.822357}{0.883856 + 0.822357} = 7.1986\%$$

2×2

$$I\%_{2\times2} = \frac{0.883856 - 0.761124}{0.822357 + 0.761124} = 7.7508\%$$

3×2

$$I\%_{3\times2} = \frac{0.822357 - 0.701718}{0.761124 + 0.701718} = 8.2469\%$$

2×3

Intentionally left blank

1×3

$$I\%_{1\times3} = \frac{0.945180 - 0.761124}{0.883856 + 0.822357 + 0.761124} = 7.4597\%$$

1×4

$$I\%_{1\times4} = \frac{0.945180 - 0.701718}{0.883856 + 0.822357 + 0.761124 + 0.701718} = 7.6825\%$$

All of the rates above are par-coupon rates.

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